Each question carries [illegible] marks.
Please answer all questions.
Time Allowed: 2hrs.

1 (a) If P, Q have continuous partial derivatives in D and $\int P\,dx + Q\,dy$ is independent of path in D then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ in D

(b) If D is simply connected and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ in D then $\int P\,dx + Q\,dy$ is independent of path in D

2 Let F be a vector field on the plane from which the origin is deleted, Write $F = (f, g)$. Assume that $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$. Let C be the circle of radius 1 centred at the origin oriented counter-clockwise. Prove that:

(i) If $\int_C F = 0$ then F has a potential function.

(ii) If $k = \frac{1}{2\pi}\int_C F$ then there exists φ such that $F = kG + \operatorname{grad}\varphi$, where

$$G(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right).$$

3. Let n_P be the unit normal vector to a surface S at P. Then

$$\left(\operatorname{Curl} F(P)\right)\cdot n_P = \lim_{r\to 0}\frac{1}{A(r)}\int_{C_r} F,$$

where for each $r>0$, C_r is the closed curve at a distance r from P on the surface S and $A(r)$ is the surface area of the region bounded by C_r.